Lecture

INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

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Subject Code:

Chapter: Unit 3 Chapter 2

Chapter Name: Models of Asset Returns

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1 Introduction

- The models developed for forecasting correlation structures fall into two categories: index models and averaging techniques.

- The most widely used technique assumes that the co-movement between stocks is due to a single common influence or index. This model is appropriately called the single-index model.

- The single-index model is used not only in estimating the correlation matrix but also in efficient market test.

- Casual observation of stock prices reveals that when the market goes up (as measured by any of the widely available stock market indexes), most stocks tend to increase in price, and when the market goes down, most stocks tend to decrease in price.

- This suggests that one reason security returns might be correlated is because of a common response to market changes, and a useful measure of this correlation might be obtained by relating the return on a stock to the return on a stock market index.

1.1 Single Factor Model

$$R_i = \alpha_{i+}\beta_i R_{M+} e_i$$

Where:

- R_i : Return on security i,
- R_M: Return on the market
- α_i, β_i : constants
- e_i : Error term, firm specific surprises, not related to the market.

1.1 Single Factor Model

$$R_i = \alpha_{i+} \beta_i R_{M+} e_i$$

- This equation simply breaks the return on a stock into two components, that part due to the market and that part independent of the market.
- Variable i in the expression measures how sensitive a stock's return is to the return on the market. A B; of 2 means that a stock's return is expected to increase (decrease) by 2% when the market increases (decreases) by 1%.
- Similarly, a B of 0.5 indicates that a stock's return is expected to increase (decrease) by 2 of 1% when the market increases (decreases) by 1%?.

1.2 Expected Return and Variance

Expected Return:

$$E_i = \alpha_i + \beta_i E_M$$

Variance :

$$V_i = \beta_i^2 V_M + V_s i$$

Where V_S is the variance of e_i

1.3 Interpreting the equation for Variance

$$V_i = \beta_i^{\,2} V_M + V_s i$$

- This equation models the variance of the return on security i as the sum of two terms.
- The first term captures the variability of return that is related to the variance of the return on the market. This is called systematic risk.
- The second term captures the variability of return that is specific to security i. This is called specific risk.

1.4 Assumptions of Single Index Model

- All relevant economic factors are summarized by one macroeconomic indicator and it moves the security market as a whole.
- beyond this common effect, all remaining uncertainty in stock returns is firm specific – there is no other source of correlation between securities.
- firm-specific events would include new inventions, deaths of key employees, and other factors that affect the fortune of the individual firm without affecting the broad economy in a measurable way.

1.5 Advantages of the Single Index Model

2 Systematic risk and specific risk

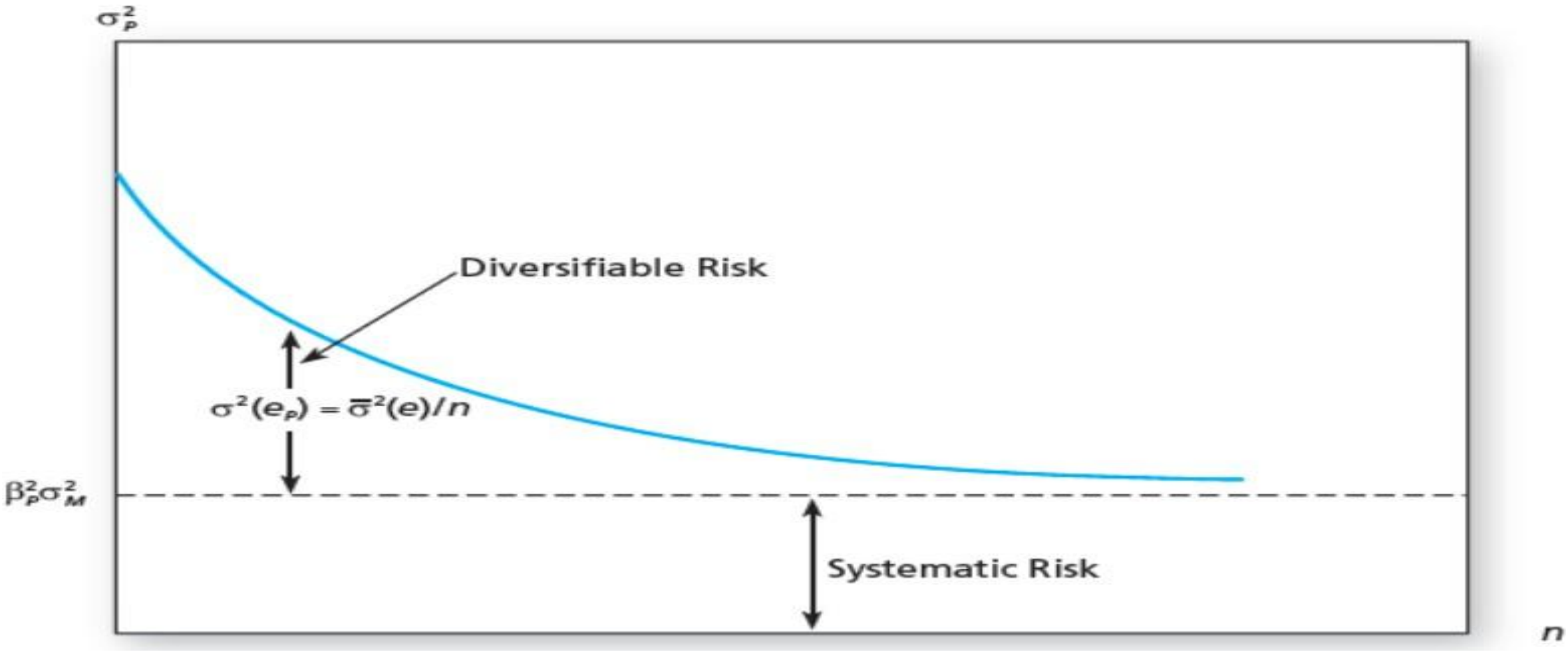
Systematic Risk is the uncertain element of the security's return related to the market as a whole.
It cannot be diversified away.

For more information, check out the article on Investopedia.

Specific Risk is the uncertain element of the security's return that depends on factors peculiar to the individual security.
It can be diversified away.

For more information, check out the article on Investopedia.

3 Variance of an Equally Weighted Portfolio



X-axis has number of securities and Y-axis has portfolio variance.
Thus, as number of securities increase, the portfolio variance decreases.

4 Covariance

Covariance

$$C_{ij} = \beta_i \beta_j V_M$$

- any correlation between the returns on two securities comes only from their joint correlation with the market as a whole.
- In other words, the only reason that securities move together is a common response to market

  movements; there are no other possible common factors.

5 Reduction in number of data items

- The use of the single-index model dramatically reduces the amount of data required as input to the portfolio selection process.

- MVPT requires estimates of N means, N variances and N(N-1)/2 covariances.

- The single-index model requires estimates of N $\alpha' s$, N $\beta' s$ and N $V' s$, plus the mean and variance of market returns.

- So, compared to mean-variance portfolio theory, the number of data items required has been reduced from N* (N+ 3)/ 2 to 3N + 2.

6 Multifactor Model

- A multi-factor model is a financial model that employs **multiple factors** in its calculations to explain asset prices.
- These models introduce **uncertainty** stemming from multiple sources.
- Multifactor models can be used to calculate the **required rate** of return for portfolios as well as individual stocks.
- The market factor can be split up even further into different **macroeconomic** factors. These may include inflation, interest rates, business cycle uncertainty.

6 Multifactor Model

A multifactor model of security returns attempts to explain the observed historical return by an equation of the form:

$$R_i = a_i + b_{i,1} I_1 + b_{i,2} I_2 + \cdots + b_{i,L} I_L + c\_i$$

Where,

- R_i is the return on security i
- a_i ,c_i are the constant and random parts respectively of the component of Return unique to security i
- I_1 ,I_2 ,...I_L are the changes in a set of L factors using which explain the variation of R_i about the expected return a_i
- b_i is the sensitivity of security i to factor k.

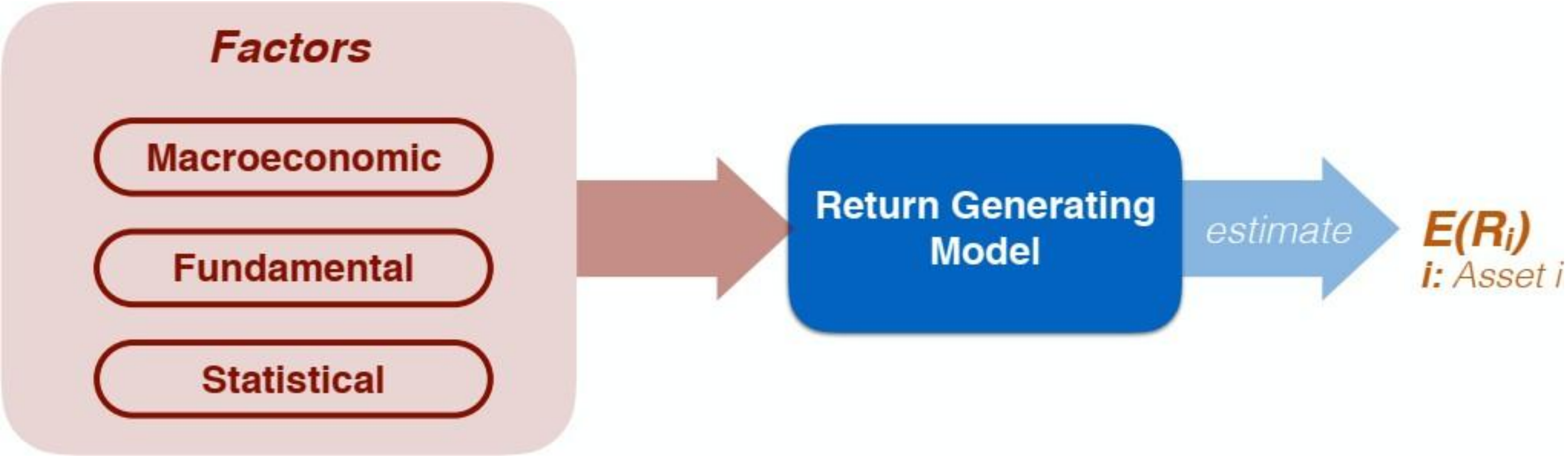
6.1 Types of Multifactor Models

- the factors are the main macroeconomic variables such as interest rates, inflation, economic growth and exchange rates.

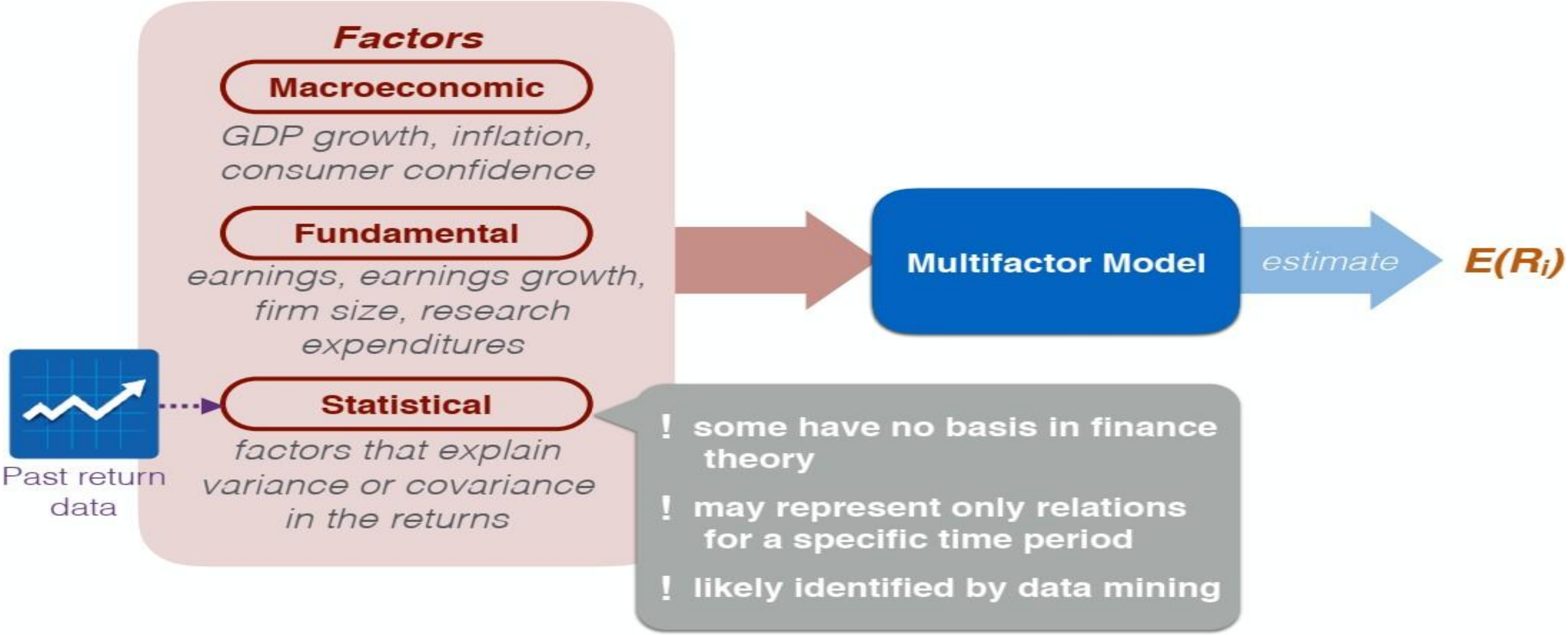
- the factors will be company specifics such as P/E ratios, liquidity ratios and gearing measurements

- the factors are not specific items initially. The method uses *principal components analysis* and historical returns on stocks to decide upon the factors.

6.1 Types of Multifactor Models



6.1 Multifactor Models



Question

CT8, April 2017, Q.9

Let Ri denote the return on security i in a two-factor model.

(i) Write down the return equation for this two-factor model, defining all additional notation that you use.

(ii) Describe the three main categories of multifactor models.

Solution

Answer:

i)

$$R_i = a_i + b_{i,1} I_1 + b_{i,2} I_2 + c_i$$

where ,

- a_i and c_i are the constant and random parts respectively of the component of the return unique to security i
- I_1 ,I_2 are the changes in a set of the two indices
- $B_{i,k}$ is the sensitivity (factor beta) of security i to factor k

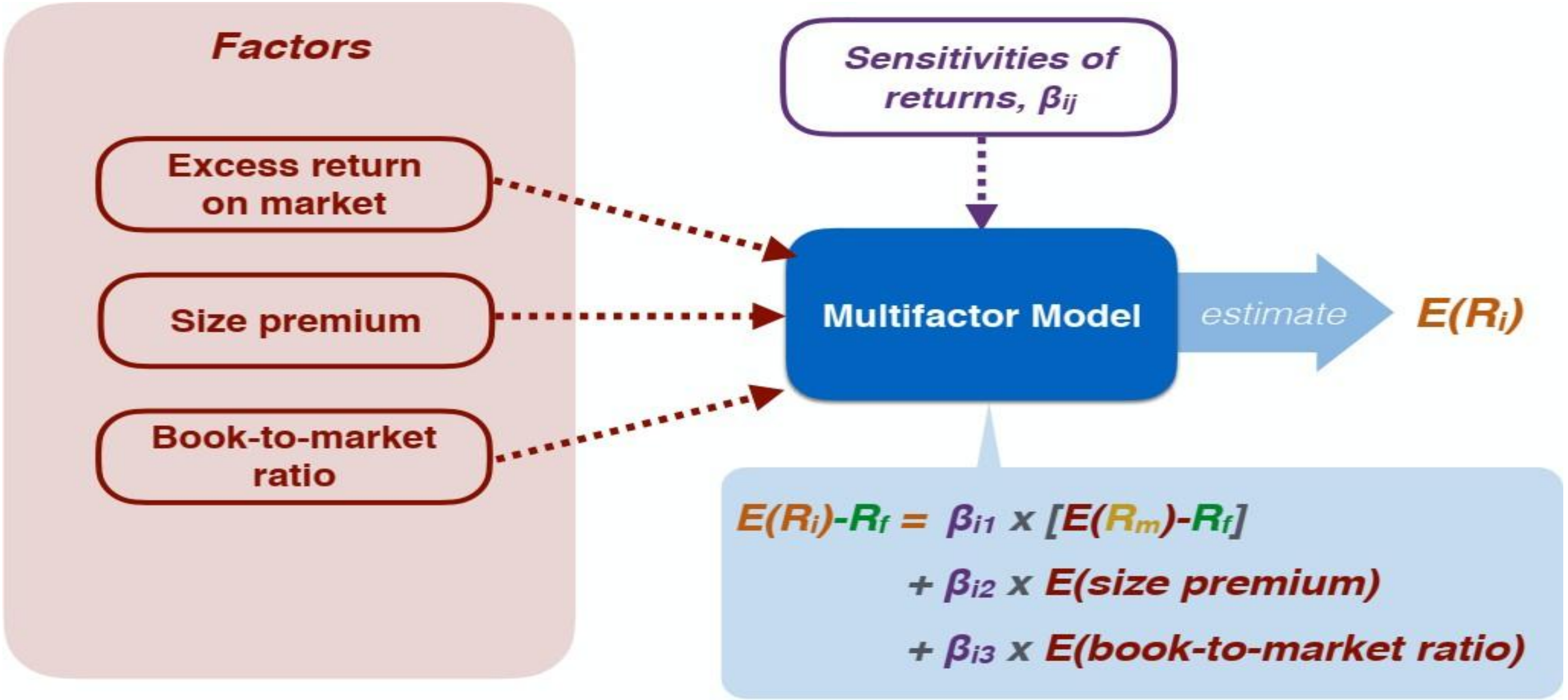
Solution

(ii) Macroeconomic factor models
These use observable economic time series as the factors, such as the annual rates of inflation and economic growth, short term interest rates, the yields on long term government bonds, and the yield margin on corporate bonds over government bonds.

Fundamental factor models
These use company specific variables as the factors, e.g. the level of gearing, the price earnings ratio, the level of research and development spending, the industry group to which the company belongs. [1]

Statistical factor models
These do not rely on specifying the factors independently of the historical returns data. Instead a technique called principal components analysis can be used to determine a set of indices which explain as much as possible of the observed variance. [1]

7 Fama and French Model



8 Which model is better at predicting returns

Although many studies have found that incorporating more factors into the model (for example industry indices) leads to better explanation of the historical data, correlation with the market is the largest factor in explaining security price valuation.

There is little evidence that multifactor models are significantly better at forecasting the future correlation structure.

Summary

- The single-index model expresses the return on a security as:

$$r_i = \alpha_i + \beta_i R_M + e_i$$

- The expected return and variance of return on security *i* and the covariance of the returns on securities *i* and *j* are given by:

$$E_i = \alpha_i + \beta_i E_M$$

$$V_i = \beta_i^2 V_M + V_{e_i}$$

$$C_{i,j} = \beta_i . \beta_j . V_M$$

- Systematic risk can be regarded as relating to the market as a whole, while specific risk depends on factors peculiar to the individual security

Summary

- The use of the single-index model dramatically reduces the amount of data required as input to the portfolio selection process.

- For N securities, the number of data items needed has been reduced from $N(N + 3)/2$ to $3N + 2$. Furthermore, the nature of the estimates required from security analysts conforms much more closely to the way in which they traditionally work.

- The goal of the builders of Multifactor models is to find a set of factors which explain as much as possible of the observed historical variation, without introducing too much 'noise' into predictions of future returns.

- Multifactor models can be classified into three categories, depending on the type of factors used.
  - Macroeconomic factor models
  - Fundamental factor models
  - Statistical factor models

- Multifactor models are significantly better at forecasting the future correlation structure.

Question

CT8, October 2016 ,Q.4

R denote the return on security i given by the following multifactor model:

$$R_i = a_i + b_{i,1} I_1 + b_{i,2} I_2 + \ldots + b_{i,L} I_L + c_i$$

Where a_i and c_i are the constant and random parts respectively of the component of the return unique to security i, $I_{1.....}$ I_L are the changes in a set of the L indices and , $b_{i,k}$ is the sensitivity (factor beta) of security i to factor k.

(i) State the category of the above model where:

(a) index 1 is a price index, index 2 is the yield on government bonds and index 3 is the annual rate of economic growth.

(b) index 1 is the level of Research and Development expenditure, index 2 is the price earnings ratio, index 3 is the level of gearing.

Question

Consider the following two-factor model for the returns on three assets A, B and C:

Asset	*A*	*B*	*C*
a_i	0.03	0.05	0.1
$b_{i,1}$	1	3	1.5
$b_{i,2}$	–4	2	1.5

(ii) Determine the equation for the expected return on a portfolio which:

(a) equally weights the three securities.
(b) has weights $x_A = -0.5$, $x_B = 1.5$, $x_C = 0$

(iii) Construct a portfolio of securities A, B, C that has a factor beta of 2 on the first factor and 1 on the second factor, i.e. the expected return on the portfolio is:

$$R_P = a_P + 2I_1 + I_2 + c_P.$$

Solution

Answer:

(i) (a) Macroeconomic.
(b) Fundamental.

Solution

(ii) The results follow from the fact that the factor beta of a portfolio on a given factor is the portfolio-weighted average of the individual securities' betas on that factor. This also applies to the constant and the random part.

Asset	*A*	*B*	*C*
a_i	0.03	0.05	0.1
$b_{i,1}$	1	3	1.5
$b_{i,2}$	–4	2	1.5
Weights			
*P*1	0.33	0.33	0.33
*P*2	–0.5	1.5	0

	*P*1	*P*2
a_P	0.06	0.06
$b_{P,1}$	1.83	4
$b_{P,2}$	–0.17	5

Solution

Hence:

(a) $E_P = 0.06 + 1.83E[I_1] - 0.17E[I_2] + E[c_P]$
for $c_P = (c_A + c_B + c_C)/3$.

(b) $E_p = 0.06 + 4E[I_1] + 5E[I_2] + E[c_P]$
for $c_P = -0.5c_A + 1.5c_B$.

(iii) The required portfolio has weights such that the portfolio-weighted averages of the betas equal to the target beta. Hence, we need to solve the linear system:

$$\begin{bmatrix} 1 & 3 & 1.5 \\ -4 & 2 & 1.5 \end{bmatrix} \begin{pmatrix} x_A \\ x_B \\ x_C \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

which returns solution $x_A = 1/8$, $x_B = 3/8$ and $x_C = 1/2$ (recall that the weights sum up to 1).

Question

CT8, October 2015, Q.4

A two-index APT model has been built, using a market (traded) index, IM, and a currency index, IC . The model for expected returns is:

$$E_i = \lambda_0 + b_{i,M}\,\lambda_M + b_{i,C}\,\lambda_C,$$

where:

- E_i is the expected return on traded security i.
- λ_M and λ_C are the expected returns on the market and currency indices respectively.
- $b_{i,M}$ and $b_{i,C}$ are the sensitivities of the returns of security i with respect to the market and currency indices respectively.

(ii) Justify why λ_0 must be 0.

Question

Suppose that a traded security has return R and the market has the following characteristics:
$Cov(R,I_M) = 0.02$.
$Var(I_M) = 0.04$.
$Var(I_C) = 0.01$.
The correlation between I_M and I_C is $Corr(I_M, I_C) = -0.4$.
The expected return on the security is $E_R = 0.09$.
$\lambda_M = 0.07$ and $\lambda_C = 0.02$.

(iii) Calculate $b_{i,M}$ and $b_{i,C}$.
(iv) Determine $Cov(R,I_C)$.

Solution

Answer:

(ii) Since IM is a traded index it must satisfy the formula (**). But the portfolio consisting of just the index has $b_{M,M}$ = 1 and $b_{M,C}$ = 0 and has expected return λ_M so we must have λ_0 =0

(iii) We must have

$R = b_{i,M} I_M + b_{i,C} I_C + c_i$, where c_i is independent of I_M and I_C.

So,

$\mathrm{Cov}(R, I_M) = b_{i,M}\mathrm{Var}(I_M) + b_{i,C}\,\mathrm{Cov}(I_M, I_C) = 0.04 b_{i,M} - 0.4 * 0.01\, b_{i,C}$
$= 0.02,$

while $E_i = b_{i,M}\lambda_M + b_{i,C}\lambda_C = 0.07\, b_{i,M} + 0.02 b_{i,C} = 0.09$

so $b_{i,M} = 0.8235$ and $b_{i,C} = 1.6176$.

Solution

(iv) $Cov(R, I_C) = b_{i,M}\ Cov(I_M, I_C) + b_{i,C}\ Var(I_C)$

$= 0.8235 * -0.008 + 1.6176 * 0.01$

$= 0.0096.$

Thank You